Rocket exhaust impingement on a ground surface

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The impingement of a rocket jet, exhausting into a vacuum, onto a solid surface is examined using the Newtonian approximation to analyse the transonic region. The jet exhaust into an unbounded vacuum required for this analysis is simplified as a quasi-radial flow using an exact calculation as a basis. Two examples are calculated giving the shock detachment distance and the ground pressures as a function of the rocket altitude.

1. Introduction

We shall be concerned here with a rocket jet exhausting into a vacuum and interacting with an infinite ground plane, oriented normal to the jet—a problem which may be of pertinence to soft lunar landings. The determination of the resulting flow field is a complex one, and as a preliminary study we shall investigate the idealized case in which the flow is assumed to be inviscid, non-conducting, and non-reacting.

The resultant flow pattern to be expected is shown schematically in figure 1. Here the jet expands abruptly at the rocket exit into a plume bounded by a straight boundary at zero pressure. Despite the apparent wide dispersal of the jet, the bulk of the exhaust mass will impinge on the ground. Near the ground a strong shock wave will form redirecting the jet along it. The location of the shock wave is not known in advance, but must be determined from the solution. The flow will be supersonic everywhere except in the imbedded subsonic domain bounded by DOACD. The flow upstream of the shock wave will thus not be influenced by the presence of the ground and will be identical to a jet flow expanding into a boundless vacuum. As a consequence we may divide the problem into two parts. The first is the determination of the jet exhaust into an unbounded vacuum—a straightforward problem which may be calculated by the method of characteristics with initial conditions at the rocket exit and the vacuum condition at the plume boundary. The second problem is the calculation of the flow in the transonic region BCDOAB. For the determination of this mixed subsonic-supersonic region, we must prescribe in addition to the flow symmetry condition along DO, and the flow tangency condition along the ground OA, the pertinent dependent variables along the downstream side of the shock segment DCB. A formal difficulty arises in an explicit formulation of the above problem because of our *a priori* ignorance of the shock location. We shall see in the next section that in the Newtonian approach this will not offer any difficulty. The calculation of the remaining supersonic region downstream of the limiting Mach wave is a purely supersonic problem which may be constructed by the method of characteristics using the transonic solution and the jet plume solution as initial conditions. In the present paper we shall omit this calculation since the primary interest will be in the transonic problem which we shall next formulate in detail.

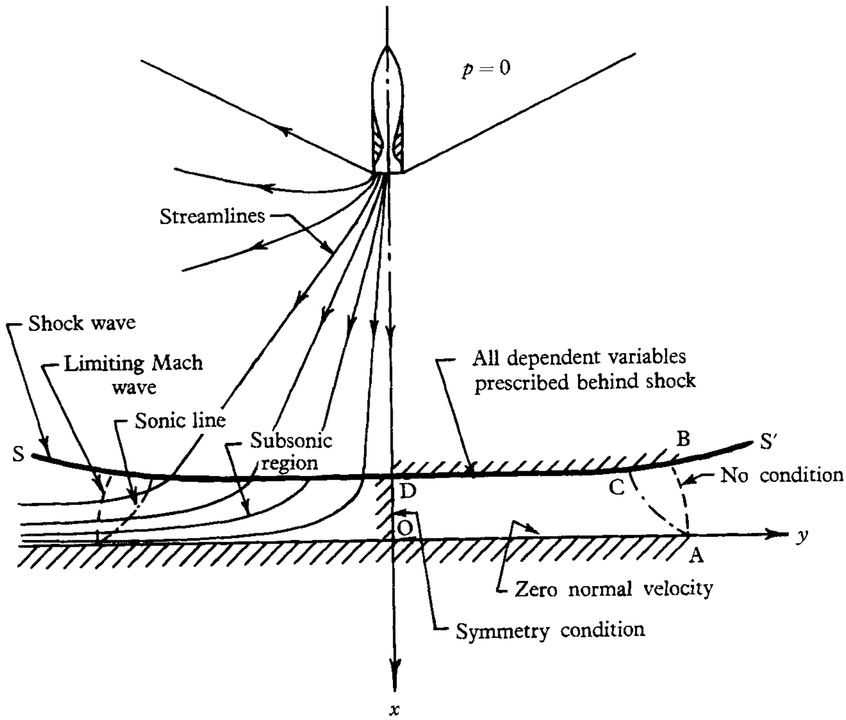

FIGURE 1. Flow pattern and boundary conditions for the transonic problem.

2. The transonic problem

The pertinent axial symmetric flow equations for the present problems are as follows:

$$\nabla.(\bar{y}\bar{\rho}\mathbf{w}) = 0, \tag{1}$$

$$\bar{\rho}\mathbf{w}.\nabla\mathbf{w} = -\nabla\bar{p}, \tag{2}$$

$$\mathbf{w}.\nabla(\bar{p}\bar{\rho}^{-\gamma}) = 0. \tag{3}$$

Here γ is the ratio of specific heats, $\bar{y}$ is the radial co-ordinate, and

$$\mathbf{w} = \tilde{\mathbf{w}}/\tilde{w}_0, \quad \bar{\rho} = \tilde{\rho}/\tilde{\rho}_0, \quad \bar{p} = (\tilde{p}-\tilde{p}_0)/\tilde{\rho}_0\tilde{w}_0^2,$$

where $\tilde{\mathbf{w}}$, $\tilde{\rho}$, and $\tilde{p}$ are the velocity vector, density, and pressure, respectively, while the subscript zero denotes reference values corresponding to a point on the axis of symmetry just upstream of the shock.

We seek now asymptotic forms of the above equations in the limit as $\epsilon = (\gamma-1)/(\gamma+1) \to 0$ using the transformations (see Freeman 1956 and Cole 1961)

$$\left.\begin{aligned} &x = \bar{x}/\epsilon^{\frac{1}{2}}, \quad y = \bar{y}, \\ &\bar{u}(\bar{x},\bar{y}) = \epsilon[u(x,y)+O(\epsilon)], \quad \bar{v}(\bar{x},\bar{y}) = \epsilon^{\frac{1}{2}}[v(x,y)+O(\epsilon)], \\ &\bar{p}(\bar{x},\bar{y}) = 1+\epsilon[p(x,y)+O(\epsilon)], \quad \bar{\rho}(\bar{x},\bar{y}) = \frac{1}{\epsilon}[\rho(x,y)+O(\epsilon)], \end{aligned}\right\} \tag{4}$$

where the $(\bar{x},\bar{y})$-co-ordinate system is shown in figure 1, and $\bar{u}$ and $\bar{v}$ are respectively the axial and radial velocity components. Introducing these transformations into the basic equations, we obtain as the resulting simplified equations (for the lowest order of ϵ),

$$u_x+v_y+v/y = 0, \tag{5}$$

$$-\rho(uu_x+vu_y) = p_x, \tag{6}$$

$$uv_x+vv_y = 0, \tag{7}$$

$$u\rho_x+v\rho_y = 0. \tag{8}$$

Here the subscripts denote partial differentiation.

Consider next the simplification of the shock jump conditions. The shock wave may be expected to be approximately parallel to the ground. Thus we set the shock slope

$$\epsilon^{-\frac{1}{2}}d\bar{x}/d\bar{y} = dx/dy = \theta(y). \tag{9}$$

For the jet flow just upstream of the shock we set

$$u = 1, \quad v = \epsilon^{-\frac{1}{2}}\bar{v} = \alpha y, \quad \rho = 1, \quad \text{and} \quad M = M_0 = \text{const.}, \tag{10}$$

where M is the Mach number, and α and M_0 will depend on the axial location of the shock. In (10) the $\epsilon^{\frac{1}{2}}$ stretching of $\bar{v}$ was required to obtain a non-degenerate jump condition; the stretching is of course permissible only if the flow inclination $\bar{v}$ upstream of the shock to the sonic point of the shock (point C of figure 1) is small. The simplified jump conditions thus become

$$v_1 = \theta+\alpha y, \tag{11}$$

$$u_1 = \beta+\theta v_1, \tag{12}$$

$$\rho_1 = \beta^{-1}, \tag{13}$$

and

$$p_1 = -[\beta+(1+2\alpha y)\theta], \tag{14}$$

where $\beta = 1+(M_0^2\epsilon)^{-1}$ and the subscript 1 refers to the conditions just downstream of the shock wave. Note that in the limiting process $\epsilon \to 0$ we must now add a further requirement that $M_0 \to \infty$ such that $\beta = \text{const.}$

Following the procedure of Freeman (1956), we transform our Newtonian equations by the hydrid co-ordinate transformation

$$r = y, \quad \tfrac{1}{2}s^2 = \psi(x,y), \tag{15}$$

where ψ is the stream function defined by

$$yu = \psi_y, \quad yv = -\psi_x. \tag{16}$$

Note from the conditions upstream of the shock, one has $s = r$ at the shock wave. Setting

$$u(x,y) = u^*(r,s), \quad v(x,y) = v^*(r,s), \quad \text{etc.},$$

we obtain as the transformed equations

$$\rho_r^* = 0 \quad \text{or} \quad \rho^* = \rho^*(s), \tag{17}$$

$$v_r^* = 0 \quad \text{or} \quad v^* = v^*(s), \tag{18}$$

$$u_r^* = (r/s)p_s^*, \tag{19}$$

$$(u^*/v^*)_s = (\beta s/r^2v^*). \tag{20}$$

These remarkably simple results are essentially due to Freeman (1956).

Equation (20) can also be integrated directly with the result

$$\frac{u^*}{v^*} = \frac{\beta}{r^2}\int_r^s \frac{\bar{s}}{v(\bar{s})}\,d\bar{s} + \frac{u_1^*}{v_1^*}. \tag{21}$$

At the surface of the ground, $s = 0$, we have $u^*/v^* = 0$. If we now use this result in (21), insert the values of u_1^* and v_1^* from (11) and (12), and differentiate (21) with respect to r, we obtain

$$\frac{d\theta^*}{dr} = \frac{\theta^*}{r}\frac{(\beta + 2v_1^{*2})}{(\beta - v_1^{*2})}, \tag{22}$$

where $v_1^* = \theta^* + \alpha r$. Equation (22) thus yields a differential equation for the determination of the shock shape.

Let us solve (22). Pertinent singular points of (22) occur at $\theta^* = r = 0$, and at $\theta^* = 0$, $v_1^{*2} = \beta$. The first point corresponds to a nodal point while the second is a saddle point with one asymptote corresponding to $\theta^* = 0$.† The latter point corresponds to the sonic point on the shock wave. Since $\theta^* = 0$ is an integral of (22) between the singular points, our desired solution is thus

$$\theta^*(r) = 0, \tag{23}$$

that is our shock wave is a straight line parallel to the ground. The shock detachment distance δ can be obtained by the requirement of mass continuity near the axis of symmetry; we obtain in the original co-ordinate system,

$$\delta = \beta\epsilon^{\frac{1}{2}}/2\alpha. \tag{24}$$

The pressure distribution may be obtained from (19) by integrating with respect to s and inserting the value of u_r^* from (21). The result for the distribution along the ground is

$$p^*(r, 0) = -\tfrac{1}{2}\beta = \text{const.} \tag{25}$$

The streamline slopes may be found from (21) and are given by

$$\tan\phi = \frac{\beta(s-r)}{\alpha r^2} + \frac{u_1^*(r)}{v_1^*(r)}. \tag{26}$$

The streamlines can then be determined by integration.

3. Numerical examples

To utilize the simple results obtained in the previous section for the transonic region, we must now calculate the jet expansion into an unbounded vacuum. The method of characteristic calculations has been carried out for two cases using an IBM 7090 program developed by J. Bowyer of General Dynamics/Astronautics. In both cases, a conical nozzle of 15° half-angle is utilized with the following flow properties:

	Case A	Case B‡
Nozzle expansion ratio	40	4·56
Ratio of specific heats	1·2	1·4

† The other asymptote is given by $\theta^* = \frac{1}{2}\pi$ and will be of no interest in the present case.

‡ Results for case B were taken from Ward (1961).

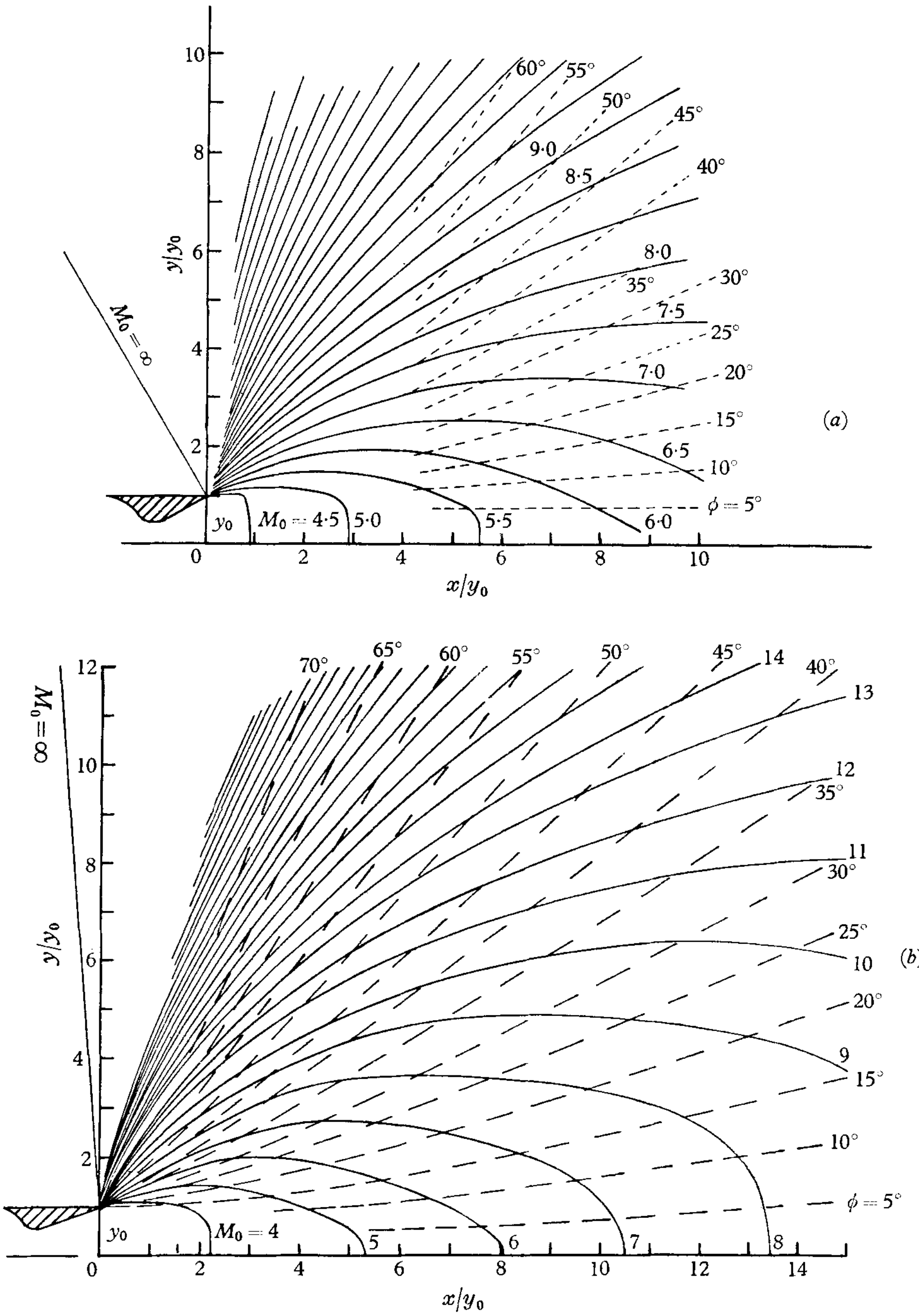

FIGURE 2. (*a*) Jet exhaust plume (case A) showing lines of constant Mach number and flow inclination. Flow exhausts to the right from the indicated nozzle. $\gamma = 1{\cdot}2$; nozzle expansion ratio $= 40$; cone half-angle $= 15°$. (*b*) Jet exhaust plume (case B) showing lines of constant Mach number and flow inclination. $\gamma = 1{\cdot}4$; nozzle expansion ratio $= 4{\cdot}56$; cone half-angle $= 15°$.

The results for the two cases are plotted in figures 2*a* and 2*b* where lines of constant Mach number and streamline slopes are plotted. In figure 3 we plot the radial variation of M_0, and the streamline slopes for three axial locations $x/r_0 = 5$, 10, 15 to check the validity of the simplified model used in (10). The location of the sonic point of the shock wave, the limit to which we use the model, is indicated for each position of the shock.

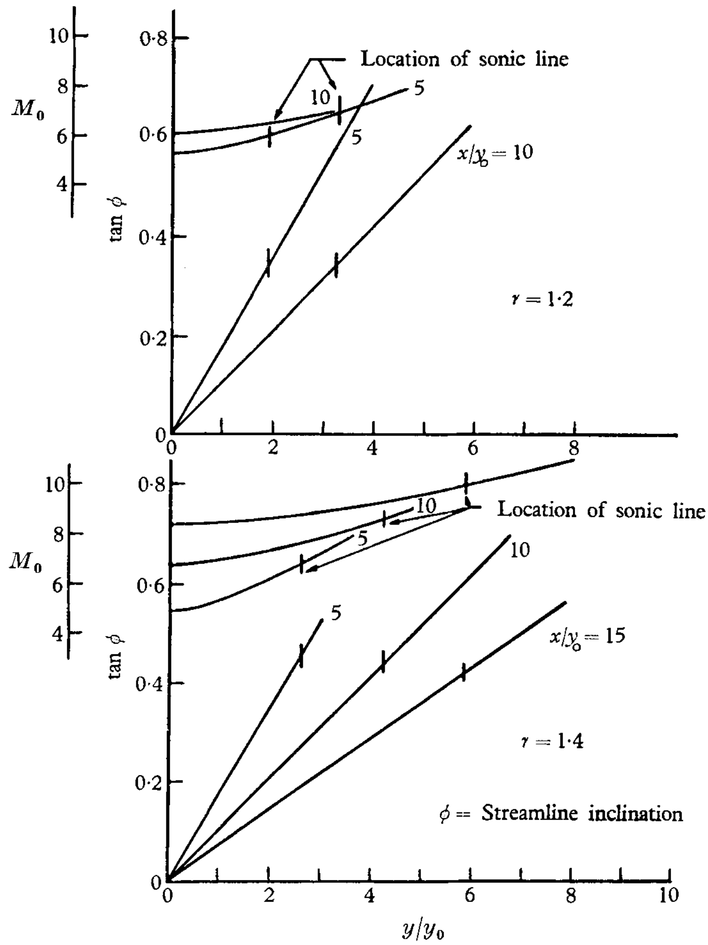

FIGURE 3. Radial variation of the Mach number and streamline slopes at various axial stations for cases A and B.

From these figures, values of β and α are determined as a function of the rocket altitude, and finally in figures 4 and 5 we plot the shock detachment distance and the value of the constant pressure on the ground, all as functions of the axial location of the shock wave. The results for the larger values of x/r_0 have been obtained by assuming the flow to be radial beyond the downstream limit of the calculations; these results are indicated by the dashed lines. Also shown in figure 5 are two experimental values for the shock detachment distance as found by Stitt† for a 15° half-angle conical nozzle at an expansion ratio of 4. For the lower value of x/r_0 a crucial portion of the shock wave was obscured in the schlieren pictures so that it was necessary to extrapolate the shock shape. The remarkable agreement in the theoretical and experimental shock detachment distances must be somewhat tempered due to the known inexactness of the theory and the

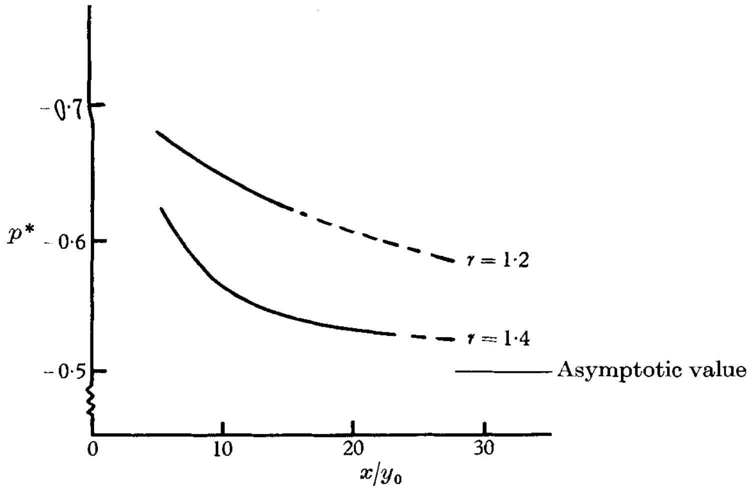

FIGURE 4. Ground pressure as a function of the axial location of the shock wave. $(\tilde{p}-\tilde{p}_0)/\tilde{\rho}_0\tilde{w}_0^2 = 1+\epsilon p^*, \quad \epsilon = (\gamma-1)/(\gamma+1)$.

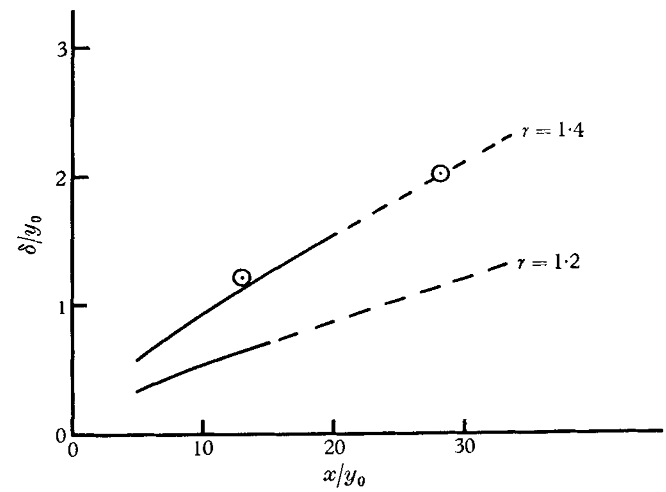

FIGURE 5. Shock detachment distance as a function of the axial location of the shock waves. ⊙, Experimental results of Stitt (1961). y_0 = nozzle exit radius.

subjective nature of interpreting the schlieren pictures. For the cases in consideration, the difference in the nozzle expansion ratios for the theoretical and experimental results should not be significant. Clearly, the present theoretical approach should give predictions of engineering value.

† The author is indebted to Mr Leonard Stitt, Head of the System Problems Section of NASA-Lewis Research Center, for furnishing enlarged schlieren pictures from experiments reported in Stitt (1961).

4. Remarks

With the use of a simplified model of the jet exhaust and the Newtonian hypothesis, an extremely simple result was obtained for the present complex flow problem. The shock wave to this approximation was found to be parallel to the ground with the detachment distance given by a simple expression in terms of the Mach number and divergence of the jet flow upstream of the shock. The ground pressure distribution was constant at the stagnation value.

The use of the simplified model of the jet flow is not essential and places a lower bound on the vehicle altitude for which we may use the resulting theory. The breakdown of the model arises primarily due to the non-constancy of the density and the Mach number along the shock wave. This defect can be readily remedied by assuming instead a parabolic radial variation of these quantities. The resulting complication in the differential equation is slight, but will require

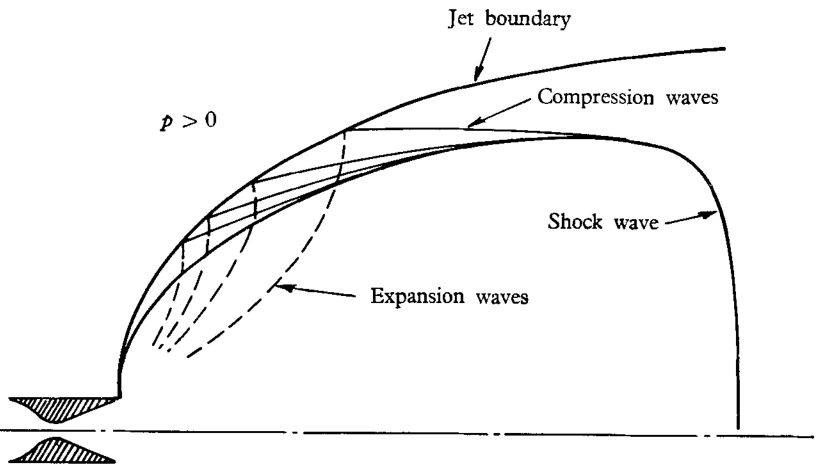

FIGURE 6. Jet exhaust into an ambient medium of small but finite pressure. Note that the location of the shock wave will depend upon the external ambient pressure.

numerical integration. The use of the Newtonian hypothesis is, on the other hand, essential, and will yield a solution valid in the subsonic region except in the vicinity of the sonic line. It is to be remembered that the uniqueness of the Newtonian solution was determined by the shock-wave characteristics at the sonic point, just as in previous applications of the Newtonian theory. This procedure can thus be justified only by an *a posteriori* empirical verification of the resulting theory.

Lastly, some comments will be of relevance regarding the modifications which might arise when the ambient pressure is non-zero but small as might be the case if one carries out an experiment of jet impingement. The expected jet exhaust into an unbounded medium for this case is shown schematically in figure 6. The essential modification here is the appearance of a shock wave which lies near the jet boundary, but which eventually reaches the axis as a normal shock wave in a Mach-disk configuration. This shock wave originates from the coalescence of compression waves produced as the result of the interactions of the left-running expansion waves from the nozzle with the constant-pressure jet-boundary. The shock wave forms a steeper angle with the streamlines than the Mach angles at points just upstream of the shock wave, so that the jet flow interior to the shock would be unaffected by the ambient pressure and will be identical with the case for zero ambient pressure at the same chamber conditions. Thus if the transonic region (including the supersonic region to the limiting Mach wave) arising from the infinite plate lies in the region interior to the shock wave, there will be no effect on the transonic flow as a result of the finiteness of the ambient pressure.

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